Numerical Simulation of the Stress-Strain Behavior of an Iron Ore Tailings from the *Quadrilátero Ferrífero*

André de Oliveira Faria
Geotechnical Engineer, Pimenta de Ávila, Belo Horizonte, Brazil, ofaria.andre@gmail.com

Mauro Pio dos Santos Júnior
Geotechnical Cordinator, Pimenta de Ávila, Belo Horizonte, Brazil, mauro.junior@pimentadeavila.com.br

Bruno Guimarães Delgado
Corporate Geotechnical Manager, Vale, Belo Horizonte, Brazil, bruno.delgado@vale.com

Jessé Joabe Vieira Carneiro
Technical Leader, Pimenta de Ávila, Belo Horizonte, Brazil, jesse.carneiro@pimentadeavila.com.br

Guilherme Henrique da Silva Pinto
Geotechnical Engineer, Pimenta de Ávila, Belo Horizonte, Brazil,
guilherme.henrique@pimentadeavila.com.br

ABSTRACT: Recent tailings dam failures indicated the need to develop careful stress-strain analysis in the Brazilian mining industry, especially for structures with brittle behavior. In this context, Critical State Soil Mechanics can be considered a useful approach to better reproduce the material behavior, given the use of the critical state concept. The objective of the present paper is to promote a discussion about the numerical simulation of the stress-strain behavior of an iron ore tailings of the *Quadrilátero Ferrífero*, considering the calibration of drained triaxial tests using two constitutive models: i) Mohr-Coulomb, widely used in Brazil; and ii) NorSand, which considers the critical state theory in its formulation. GeoStudio (SIGMA/W) software was used to develop the numerical simulation, as well as the data available in the literature to define the material geotechnical characterization was used. About the results, the Mohr-Coulomb model confirmed its limitations in terms of perfectly plastic response, while NorSand presented the best calibration, which can be associated with the use of the largest number of parameters, the application of the state parameter concept, and the silty nature of the tailings analyzed. Additionally, it was observed difficulty in simulating the material dense behavior.

KEYWORDS: Critical State Soil Mechanics, Mohr-Coulomb, NorSand

RESUMO: Eventos de ruptura recentes em barragens de rejeitos indicaram a necessidade de desenvolver análises tensão-deformação cuidadosas na indústria de mineração brasileira, especialmente para estruturas com comportamento frágil. Neste contexto, a Mecânica dos Solos dos Estados Críticos pode ser considerada uma abordagem útil para melhor reproduzir o comportamento do material, dada a utilização do conceito de estado crítico. O objetivo do presente artigo é promover uma discussão sobre a simulação numérica do comportamento tensão-deformação de um rejeito de minério de ferro do Quadrilátero Ferrífero, considerando a calibração de ensaios triaxiais drenados utilizando dois modelos constitutivos: i) Mohr-Coulomb, amplamente utilizado no Brasil; e ii) NorSand, que considera a teoria do estado crítico em sua formulação. O software GeoStudio (SIGMA/W) foi utilizado para desenvolver a simulação numérica, bem como os dados disponíveis na literatura para definir a caracterização geotécnica do material foram utilizados. Sobre os resultados, o modelo Mohr-Coulomb confirmou suas limitações em termos de resposta perfeitamente plástica, enquanto o NorSand apresentou a melhor calibração, o que pode estar associado à utilização do maior número de parâmetros, à aplicação do conceito de parâmetro de estado e à natureza siltosa do rejeito analisado. Adicionalmente, foi observada dificuldade em simular o comportamento denso do material.

PALAVRAS-CHAVE: Mecânica dos Solos dos Estados Críticos, Mohr-Coulomb, NorSand

1 INTRODUCTION

Stresses and strains can be related through constitutive equations so that the choice of the appropriate solution depends on the degree of approximation to describe the real behavior of the material. The greater this approximation, the greater the complexity of the relationship and, consequently, more parameters related to the mechanical properties of the material.

The Critical State Soil Mechanics (CSSM) proposes an integrated approach to material mechanical behavior, introducing the concept of critical state line (CSL) and using the correlation between resistance and deformability. Taylor (1948) was the first author to report that the critical void ratio is affected by mean effective stress, becoming smaller as the stress level increases. This relationship between the critical void ratio and the mean effective stress was called as CSL, which can be described on the $q – p' – e$ surface and separately on the $q – p'$ and $e – p'$ planes, as shown by Equation 1 and 2, respectively.

$$q = Mp' \tag{1}$$

$$e_c = \Gamma - \lambda \ln (p'_c) \tag{2}$$

Where q is the deviator stress, M is the critical friction ratio, Γ is the 'altitude' of CSL defined at 1 kPa, λ is the slope of CSL and p'_c is the critical mean effective stress.

Recent tailings dam failures in Brazil, namely Brumadinho B-I Dam (2019) and Fundão Dam (2015), indicated the relevance of assessing the concepts of the critical state theory. Additionally, studies of the behavior of mining tailings adopting these concepts are recent (Smith et al., 2019; Silva et al., 2022; Carneiro et al., 2023; Costa et al., 2024), which illustrates that research about this theme is restricted.

To promote a discussion about the need to use the concepts of CSSM in stress-strain studies, in addition to classical soil mechanics, considering the context of the state of practice in the Brazilian mining industry, the present paper focuses on the assessment of the numerical simulation of the stress-strain behavior of an iron ore tailings from the *Quadrilátero Ferrífero* (Minas Gerais state, Brazil) and promote a comparison of the numerical results with the experimentally triaxial tests used. For this, it was considered the i) Mohr-Coulomb constitutive model, which does not adopt the critical state theory and is widely used in the Brazilian mining industry; and ii) the NorSand model, which is based on CSSM.

2 GEOTECHNICAL CHARACTERIZATION

The work was conducted using the data available in the literature indicated by Eloi (2021), who developed a campaign of tests to discuss conventional and advanced laboratory procedures to evaluate the CSL in iron ore tailings, considering the performance of triaxial tests in drained conditions. According to the author, the data came from tests performed in the Geotechnical Laboratory (LabGeo) of the Faculty of Engineering of the University of Porto (FEUP).

The geotechnical characterization was defined by using i) the grain size distribution (LNEC E 196, 1966); ii) the specific gravity (Gs) considering the Portuguese Standard NP-83 (1965); and iii) drained triaxial tests with the specimens consolidated to 100 kPa, 200 kPa and 400 kPa. The tests performed to evaluate the Atterberg limits characterized the material as non-plastic (Eloi, 2021).

The grain size distribution curve (ASTM D422-63) indicated that the average material composition is 8.1% of sand, 83.6% of silt, and 8.3% of clay (Figure 1), with an average specific gravity (Gs) of 4.55.

Four drained isotropically consolidated triaxial compression tests were conducted with the iron ore tailings in: i) loose state (CID-01, CID-02, and CID-04); and ii) dense state (CID-03). The maximum deviatoric criterion was used to define the failure surface and were obtained the parameters of c' = 0 kPa and φ' = 35.4°.

The definition of the CSL was possible using the CID-01, CID-02 and CID-04 tests, as shown in Figure 2 (M = 1.397, $\varphi'_c = 34.5°$, λ = 0.037 and Γ = 1.038). Considering that the CID-03 test did not reach the CSL, which can occur by the dense condition of the tailings during the performance of the test and, consequently, by the possible formation of 'shear bands', it was not used to define the CSL. Also, according to ICOLD (2022), the difficulty of the CID-03 to reach the critical state can be associated with the distance of the void ratio of the CSL, since the further the void ratio is from the CSL, the greater the amount of volume change that needs to occur to reach the critical state.

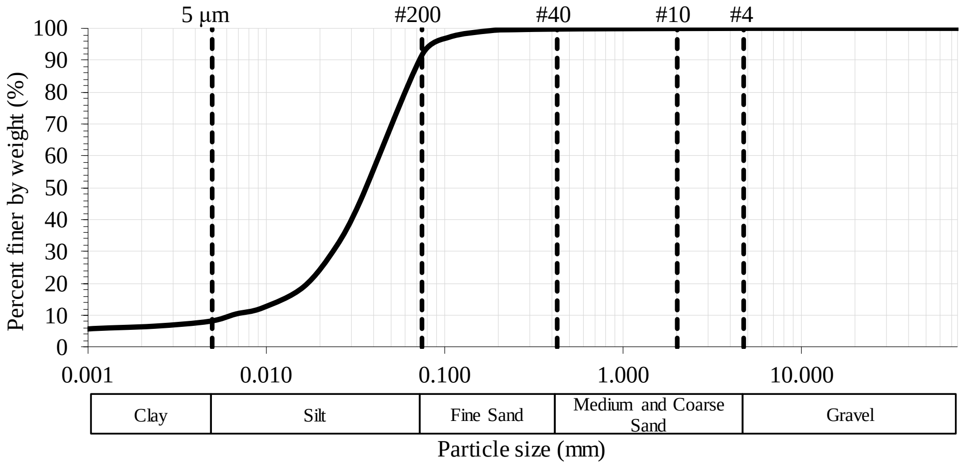

Figure 1 – Particle Size Distribution (PSD) according to the standard D422-63.

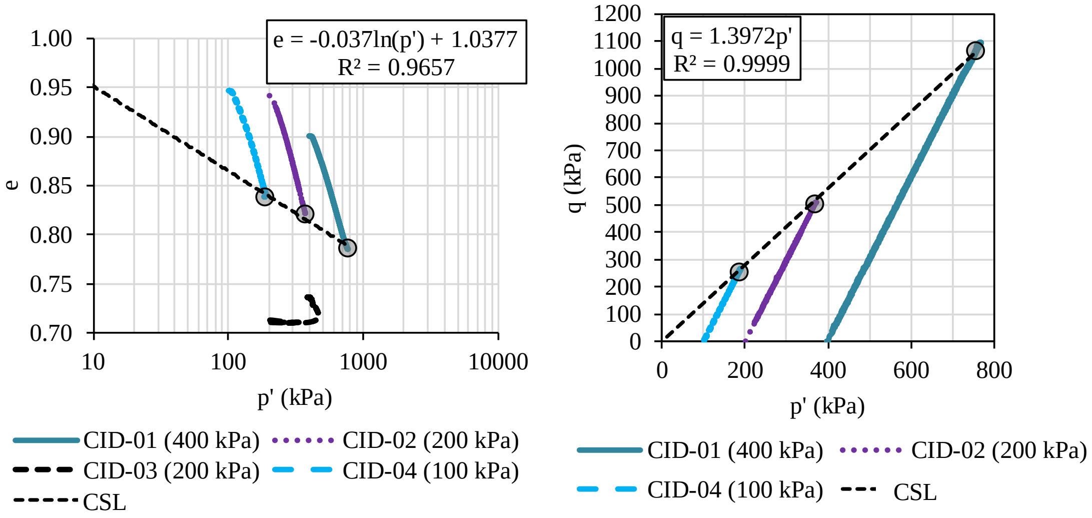

Figure 2 – CSL on the $e-p'$ and $q-p'$ planes.

3 CONSTITUTIVE MODELS

3.1 Mohr-Coulomb

The Mohr-Coulomb elastoplastic model uses a linear relationship between the shear stress (τ) and the effective normal stress (σ'), suggesting that the yield begins when the shear stress satisfies Equation 3.

$$\tau = c' + \sigma' \tan(\varphi') \tag{3}$$

Where c' is the effective cohesion intercept and φ' is the effective friction angle. The yield function, in terms of principal stress, is given as shown in Equation 4.

$$F_s(\sigma) = (\sigma_1 - \sigma_3) - (\sigma_1 + \sigma_3)\sin(\varphi') - 2\cos\varphi' c' \tag{4}$$

Where σ_1 is the major principal stress and σ_3 is the minor principal stress. The yield function represents a pyramid of hexagonal cross section in the space of principal stress, which is fixed in the space of stress and does not change in size with additional plastic deformations. Thus, the model is normally described as perfectly plastic.

One of the limitations of this model is that the Mohr-Coulomb parameters are only associated with the strength of the soil, while it is also important to consider the behavior in terms of deformability/compressibility. Besides that, the perfectly plastic response becomes unfeasible since the volumetric deformation related to the plastic portion continues to develop indefinitely with additional shear, when the dilation angle (ψ) is greater than zero.

Table 1 presents the parameters defined after characterization to model the iron ore tailings with Mohr-Coulomb. Different values were evaluated for the effective Young's modulus (E') and Poisson's ratio (v), to obtain the best calibration for the analysis. Additionally, the dilation angle (ψ) was defined from the slope of the curve that describes the relationship between the volumetric strain (ε_v) and shear strain (ε_s) of the tailings.

Table 1. Mohr-Coulomb model parameters.

Parameters	CID-01	CID-02	CID-03	CID-04
e_0	0.90	0.94	0.71	0.95
γ (kN/m³)	28.12	27.73	30.13	27.69
E' (MPa)	15	7	70	4
v	0.10			
c' (kPa)	0.00			
φ' (°)	35.4			
ψ (°)	-	-	2.36	-

3.2 NorSand

Adopting the critical state theory in its formulation, the NorSand model was created in the 1980s and 1990s, using the experience observed in the construction of structures on loose sands. The model was initially described by Jefferies (1993) and later by research papers didactically compiled by Jefferies & Been (2016).

While the Cam-Clay model, which was the first model to consider the CSSM, adopts the existence of a single NCL parallel to the CSL as described by Taylor (1948), the NorSand model adopts the concept of infinite NCL, which differs according to the initial void ratio of the geomaterial, and the consideration that all of them intercept the CSL (Jefferies, 1993).

Knowing that the first model to adopt the state parameter (Ψ) within the CSSM was NorSand, Jefferies & Been (2016) suggest a unique relation between the state parameter and the maximum dilatancy (D_{min}), as shown in Equation 5.

$$D_{min} = \chi_{tc}\Psi \tag{5}$$

About the plasticity parameters of the model, the volumetric coupling coefficient from Nova's flow rule (N), which is also referred as stress-dilatancy, can be defined by Equation 6 (Jefferies & Shuttle, 2002), as well as the plastic hardening modulus (H) can be defined according to the calibration with the process Iterative Forward Modeling (IFM) proposed by Jefferies & Been (2016), using the Equation 7.

$$\eta_{m\acute{a}x} = M_{tc} - (1 - N)D_{min} \tag{6}$$

$$H = H_0 - H_y\Psi \tag{7}$$

Where for the present paper the N parameter was defined according to the CID-03 (dense condition) and the H_0 is the modulus for the condition of state parameter equal to zero, as well as H_y represents the modulus as a function of the state parameter. Table 2 presents the parameters defined after characterization to model the iron ore tailings with NorSand. It is noteworthy to mention that the elastic shear modulus (G) was adjusted to obtain the best calibration, since specific tests were not performed to define this parameter with assertiveness.

Table 2 – NorSand model parameters.

Parameters	CID-01	CID-02	CID-03	CID-04
e_0	0.90	0.94	0.71	0.95
γ (kN/m^3)	28.12	27.73	30.13	27.69
OCR	1			
Ψ_0	0.08	0.10	-0.12	0.08
ν	0.10			
G_{ref} (MPa)	50			
m	0.50			
p'_{ref}(kPa)	100			
Γ	1.037			
λ	0.037			
M	1.397			
N	0.498			
χ	5.822			
H	30	40	400	40
H_0	184.5			
H_y	1,666			

4 NUMERICAL ANALYSIS METHODOLOGY

The triaxial tests were modeled using the Finite Element Method (FEM) in GeoStudio 2021.3 software (SIGMA/W module), considering the axisymmetric condition. This option allows the simplification of the model's geometry and, consequently, the optimization of the calculation time.

The analysis was performed considering two phases: i) consolidation; and ii) shear. The option of isotropic elastic material was used for the consolidation, as well as in this phase was applied the boundary conditions and confining stress of 100 kPa, 200 kPa, and 400 kPa. In the sequence, the displacements and strains were reset, and the shear phase was initiated with the materials defined by the two constitutive models (Mohr-Coulomb and NorSand).

A first-order horizontal, as well as vertical displacement restrictions and strain control were adopted as boundary conditions, as well as a quadratic mesh with elements of 0.01 m was used.

5 RESULTS

Figure 3 shows the numerical calibration results obtained for the Mohr-Coulomb model and the experimental triaxial tests used.

Considering the strength behavior, the model presented values slightly higher than the drained triaxial compression tests, with emphasis on the CID-02 specimen, which showed the greatest difference between the numerical and experimental stress-strain response. Except for the CID-01 specimen, the numerical response indicated volumetric strains slightly higher than the triaxial tests.

After evaluating the numerical modeling, the inability of the model to reproduce the ductile/brittle behavior of the iron ore tailings was noted, being restricted to the perfectly plastic response, which is noticed from the abrupt change in the elastic/plastic behavior at axial strains near 7%. It is important to note that these behaviors occurred due to inherent limitations of the Mohr-Coulomb model, as was discussed in section 3.1 of the present paper.

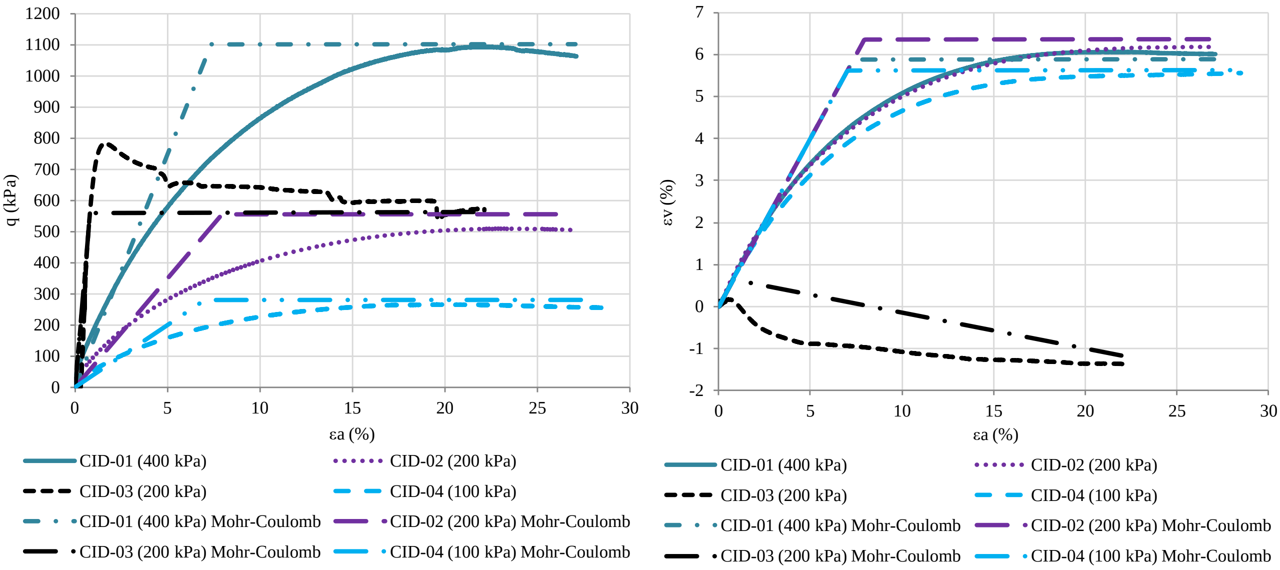

Figure 3 – Stress-strain (q - ε_a) and strain paths (ε_v - ε_a) – Mohr Coulomb model.

Despite the limitations, for many practical applications in materials with drained and contractive behavior at low strain levels, ($\varepsilon_a < 5\%$), the model preserves a good adherence to the triaxial curves (indicated on the q - ε_a plane for the CID-01, CID-02 e CID-04 tests), with the behavior approaching a perfectly elastic behavior for these stress levels. This fact has justified the wide use of the model in the Brazilian mining industry nowadays. However, for structures subjected to higher strains or undrained conditions (associated with strain-softening behavior), this approximation can be classified as an oversimplification.

Considering the modeling with the NorSand model, Figure 4 shows the numerical calibration results obtained for the model and the experimental triaxial tests used.

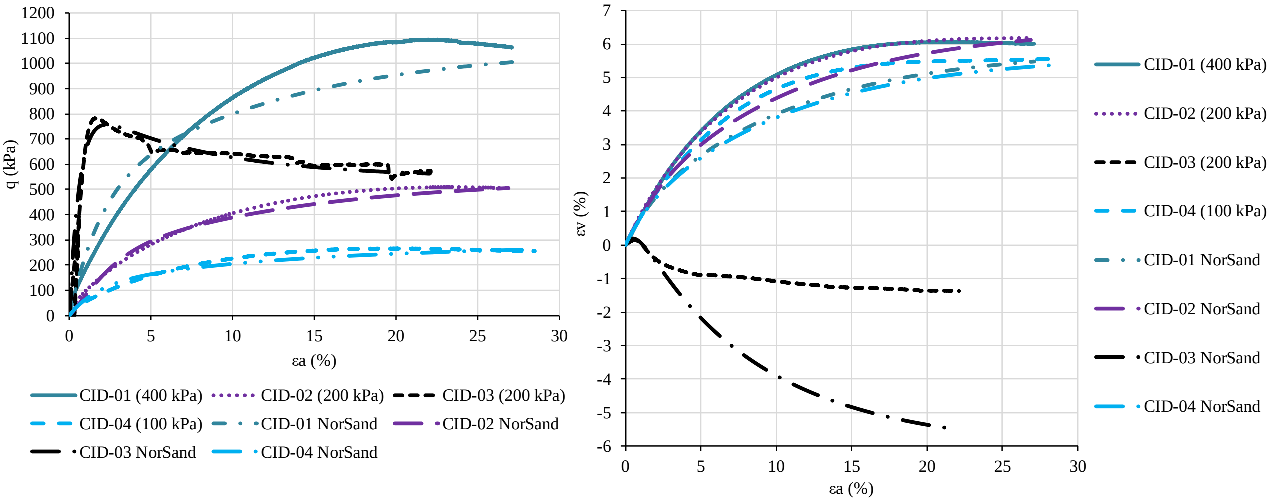

Figure 4 – Stress-strain (q - ε_a) and strain paths (ε_v - ε_a) – NorSand model.

As indicated by Figure 4, the stress-strain numerical results were close to the triaxial tests, showing that the CID-01 numerical result was the most divergent when compared with the experimental behavior. Also, the strength loss observed at low deformations ($\varepsilon_a < 5\%$) in the CID-03 test was properly simulated by NorSand, which shows that the model reproduced the ductile/brittle behavior of the iron ore tailings, differently from the Mohr-Coulomb constitutive model.

Considering the strain paths for the loose specimens, the model indicated a good adherence in general. In this context, the numerical modeling results showed that the volumetric strain for loose specimens (CID-01, CID-02, and CID-04) was slightly lower than the triaxial tests, with the higher divergence observed in the CID-01 numerical model. For the dense specimen (CID-03), the model indicated a good adherence until the peak deviator stress, while the post-peak condition did not fit the behavior indicated by the triaxial test. According to Silva et al. (2022), this divergence can be explained by the formation of 'shear bands' during the experimental tests performed with very dense samples and the difficulty of reproducing this phenomenon with numerical modeling and, consequently, with the NorSand model.

Desrues et al. (1996) evaluated the void ratio evolution inside the shear bands in triaxial sand specimens studied by computed tomography and observed the tendency of the local void ratio to stabilize after a large jump during the first stage of the localization development (shear band). These findings could suggest that if it was possible to determine the void ratio inside the possible shear band of the CID-03 test, the NorSand model could better represent the phenomenon, since the model was able to simulate the void ratio variation until the CSL (which was expected), but considering the global void ratio of $e_0 = 0.71$ (Figure 5a).

To better calibrate the CID-03 test, different parameters of the model were evaluated and concluded that the state-dilatancy (χ) was responsible for the most influence in the volumetric strains (Figure 5b). It is noteworthy to mention that the χ parameter was defined with just one test in dense condition (CID-03).

To investigate the experimental reproducibility of χ, Reid et al. (2023) performed a dilatancy round robin between five laboratories, with testing undertaken on gold tailings. The study concluded that for samples reconstituted by dense moist tamping (DMT) and vibration-densification, the variation of χ was from 4 to 12, while for tests performed with samples reconstituted by slurry deposition (SD) and the air dried (AD) the variation of χ was from 1.5 to 2.1, showing that significant variation was seen. Considering the poor reproducibility of χ between laboratories, Reid et al. (2023) suggest it is currently prudent to acknowledge uncertainty in χ for the purpose of engineering analyses of TSFs. This aspect reinforces the need for carrying out very careful tests (Viana da Fonseca et al., 2021) to seek greater assertiveness of this parameter.

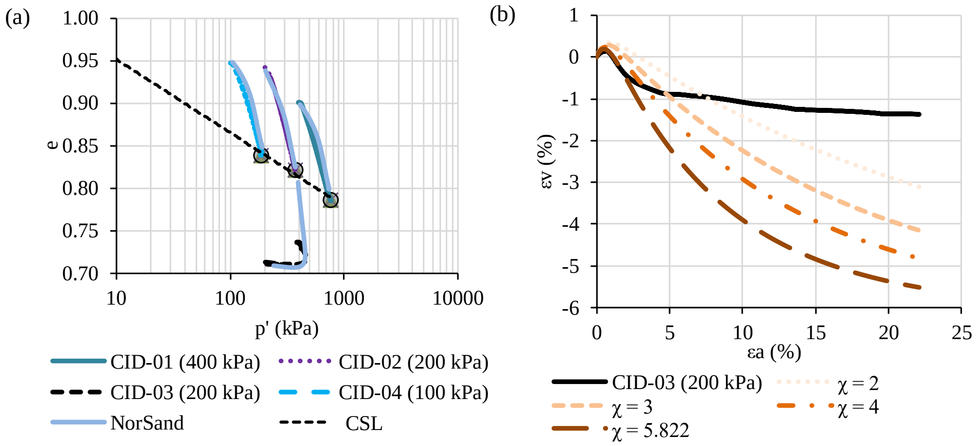

Figure 5 – (a) void ratio variation and (b) sensibility evaluation of χ – NorSand model.

6 CONCLUSIONS

This paper presented an evaluation of the behavior of an iron ore tailings using numerical models (Mohr-Coulomb and NorSand) to calibrate four experimental drained triaxial tests performed on silt tailings. Also, characterization of the tailings was presented to describe the material, which did not reach the critical state for the CID-03 test in the dense condition. The difficulty of the CID-03 to reach the CSL can be associated with the occurrence of 'shear bands' during the test and the distance of the void ratio of the CSL, since the further the void ratio is from the CSL, the greater the amount of volume change to reach it (ICOLD, 2022).

The numerical results from the use of the Mohr-Coulomb model confirmed the limitations in terms of perfectly plastic response, not being able to simulate the ductile/brittle behavior of the material. The model preserved a good adherence to the experimental curves just for behavior at low strain levels, ($\varepsilon_a < 5\%$).

The NorSand simulated the ductile/brittle behavior of the material, but the numerical results for the material in dense condition (CID-03) indicated considerably volumetric strains compared to the triaxial test, which can be associated with the occurrence of 'shear bands' during the experimental tests (Desrues et al., 1996; Silva et al., 2022). Additionally, difficulty in using the state-dilatancy (χ) was highlighted, since it was defined just with only one triaxial test and a higher quantity of tests in dense condition can provide a better calibration of this parameter, considering its possible uncertainty (Reid et al, 2023).

The results indicated by the NorSand model are good evidence to illustrate the importance of considering the concepts of the CSSM in stress-strain studies, especially in the Brazilian mining industry, in which simplified models and approaches are adopted.

ACKNOWLEDGEMENTS

The authors would like to acknowledge NUGEO/UFOP and Pimenta de Ávila Consultoria.

REFERENCES

ASTM D422-63. (2007). *Standard test method for particle-size analysis of soils*. ASTM International, West Conshohocken, PA. http://dx.doi.org/10.1520/D0422-63R07E01.

Carneiro, J.J.V., Faria, A.O., dos Santos Júnior, M.P. (2023). *Calibration of Modified Cam-Clay Parameters for Red Mud Tailings – A Case Study*. In Proceedings of Tailings and Mine Waste 2023 (pp. 69-80). Vancouver.

Costa, F.F.S., Delgado, B.G., Castilho, B.M. (2024). *Numerical sensitivity analysis for stress-strain simulation and flow liquefaction assessment of tailings storage facilities using the NorSand constitutive model*. Soils and Rocks, 47(1), 1-9. https://doi.org/10.28927/SR.2024.011322.

Desrues, J., Chambon, R., Mokni, M., Mazerolle, F. (1996). *Void ratio evolution inside shear bands in triaxial sand specimens studied by computed tomography*. Geotechnique, 46(3), 529-546. https://doi.org/10.1680/geot.1996.46.3.529.

Eloi, D.M. (2021). *Ensaios triaxiais com procedimentos convencionais e avançados para avaliação de estados críticos*. MSc dissertation, Universidade do Porto, 99 p. https://hdl.handle.net/10216/133473.

International Commission on Large Dams (ICOLD). (2022) 'Tailings Dam Safety Bulletin N° 194, Version 1.0', Paris.

Inspeção Geral dos Produtos Agrícolas e Industriais – IGPAI. (1965). NP-83: determinação da densidade das partículas. Lisboa: IGPAI (in Portuguese).

Jefferies, M. (1993). *Nor-Sand: a simple critical state model for sand*. Geotechnique, 43(1), 91-103. http://dx.doi.org/10.1680/geot.1993.43.1.91.

Jefferies, M., & Been, K. (2016). *Soil liquefaction: a critical state approach (2nd ed.)*. Boca Raton: CRC Press.

Jefferies, M., & Shuttle, D.A. (2002). *Dilatancy in general Cambridge-type models*. Geotechnique, 52(9), 625-638. http://dx.doi.org/10.1680/geot.2002.52.9.625.

Laboratório Nacional de Engenharia Civil – LNEC. (1966). *LNEC E 196: solos: análise granulométrica*. Lisboa: LNEC (in Portuguese).

Reid, D., Fourie, A., Dickinson, S., Shanmugarajah, T., Fanni, R., Smith, K., Garfias, J., Yuan, B., Ghafghazi, M. and Duyvestyn, A. (2023). *Results of a dilatancy round robin*. Canadian Geotechnical Journal. In press.

Silva, J.P.S., Cacciari, P.P., Torres, V.F.N., Ribeiro, L.F.M., & Assis, A.P. (2022). *Behavioural analysis of iron ore tailings through critical state soil mechanics*. Soils and Rocks, 45(2), 1-13. http://dx.doi.org/10.28927/SR.2022.071921.

Smith, K., Fanni, R., Chapman, P., & Reid, D. (2019). *Critical state testing of tailings: comparison between various tailings and implications for design*. In Proceedings of the Tailings and Mine Waste (pp. 1183-1195). Vancouver: University of British Columbia.

Taylor, D.W. (1948). Fundamentals of soil mechanics. New York: John Wiley & Sons.

Viana da Fonseca, A., Cordeiro, D., & Molina-Gómez, F. (2021). *Recommended procedures to assess critical state locus from triaxial tests in cohesionless remoulded samples*. Geotechnics, 1(1), 95-127. http://dx.doi.org/10.3390/geotechnics1010006.